Mid Ch. Review

(1) $\cos x\left(\tan x + \cot x\right) = \csc x$

$$\cos x \cdot \frac{\sin x}{\cos x} + \frac{\cos x \cdot \cos x}{\sin x}$$

$$\frac{\sin^2 x + \cos^2 x}{\sin x}$$

$$\frac{1}{\sin x}$$

$$\csc x$$

(2) $\dfrac{\sin(x+\pi)}{\cos\left(x+\frac{3\pi}{2}\right)} = \tan^2 x - \sec^2 x$

$$\tan^2 x - (\tan^2 x + 1)$$

$$-1$$

$$\frac{\sin x\cos\pi + \sin\pi\cos x}{\cos x\cos\frac{3\pi}{2} - \sin x\sin\frac{3\pi}{2}}$$

$$\frac{-\sin x + 0}{0 + \sin x}$$

$$-\frac{\sin x}{\sin x}$$

$$-1$$

③ $(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = 2$

$\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta = 2$

$1 + 1 = 2$

④ $\dfrac{\sin t - 1}{\cos t} = \dfrac{\cos t - \cot t}{\cos t \cot t}$

$\dfrac{\sin t}{\cos t} - \dfrac{1}{\cos t}$

$\tan t - \sec t$

$\dfrac{\cos t}{\cos t \cot t} - \dfrac{\cot t}{\cos t \cot t}$

$\dfrac{1}{\cot} - \dfrac{1}{\cos t}$

$\tan t - \sec t$

⑤ $\dfrac{1 - \cos 2x}{\sin 2x} = \tan x$

$\dfrac{1 - (1 - 2\sin^2 x)}{2\sin x \cos x}$

$\dfrac{2\sin^2 x}{2\sin x \cos x}$

$\dfrac{\sin x}{\cos x}$

$\tan x$

(7) $\frac{\sin x}{\tan x} + \frac{\cos x}{\cot x} = \sin x + \cos x$

$$\frac{\sin x}{\frac{\sin x}{\cos x}} + \frac{\cos x}{\frac{\cos x}{\sin x}}$$

$$\sin x \cdot \frac{\cos x}{\sin x} + \cos x \cdot \frac{\sin x}{\cos x}$$

$$\cos x + \sin x$$

(8) $\sin^2 \frac{x}{2}$ $\qquad \frac{\tan x - \sin x}{2 \tan x}$

$$\frac{\tan x - \sin x}{2 \frac{\sin x}{\cos x}}$$

$$\tan x \left(\frac{1}{2} \frac{\cos x}{\sin x}\right) - \sin x \left(\frac{1}{2} \frac{\cos x}{\sin x}\right.$$

$$\frac{\sin x}{\cos x} \cdot \frac{1}{2} \frac{\cos x}{\sin x}$$

$$\frac{1}{2} - \frac{1}{2} \cos x$$

$$\frac{1 - \cos x}{2}$$

$$\left(\sqrt{\frac{1 - \cos x}{2}}\right)^2$$

$$\sin^2 \frac{x}{2}$$

(9) $\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right]$

$\frac{1}{2}\left[(\sin\alpha\cos\beta + \sin\beta\cos\alpha) + (\sin\alpha\cos\beta - \sin\beta\cos\alpha)\right]$

$\frac{1}{2}(2\sin\alpha\cos\beta)$

$\sin\alpha\cos\beta$

(10) $\dfrac{1 + \frac{1}{\sin x}}{\frac{1}{\cos x}} - \dfrac{\cos x}{\sin x} = \cos x$

$\cos x\left(1 + \frac{1}{\sin x}\right) - \frac{\cos x}{\sin x}$

$\cos x + \frac{\cos x}{\sin x} - \frac{\cos x}{\sin x}$

$\cos x$

(11) $\dfrac{\cot x - 1}{\cot x + 1} = \dfrac{1 - \tan x}{1 + \tan x}$

$\dfrac{\left(\frac{\cos x}{\sin x} - 1\right)\cdot\sin x}{\left(\frac{\cos x}{\sin x} + 1\right)\sin x}$ $\qquad$ $\dfrac{\left(1 - \frac{\sin x}{\cos x}\right)\cdot\cos x}{\left(1 + \frac{\sin x}{\cos x}\right)\cos x}$

$\dfrac{\cos x - \sin x}{\cos x + \sin x}$ $\qquad$ $\dfrac{\cos x - \sin x}{\cos x + \sin x}$

(12) $2\sin^3\theta\cos\theta + 2\sin\theta\cos^3\theta = \sin 2\theta$

$2\sin\theta\cos\theta(\sin^2\theta + \cos^2\theta)$

$2\sin\theta\cos\theta(1)$

$\sin 2\theta$

(13) $\dfrac{\sin x + \cos x}{\sec x + \csc x} = \dfrac{\sin x}{\sec x}$

$$\frac{\sin x + \cos x}{\frac{1}{\cos x} + \frac{1}{\sin x}}$$

$$\frac{\sin x + \cos x}{\frac{\sin x + \cos x}{\sin x \cdot \cos x}}$$

$$\sin x + \cos x \cdot \frac{\sin x \cos x}{\sin x + \cos x}$$

$\sin x \cos x$

$\dfrac{\sin x}{\sec x}$

(14) $\sec 2x = \dfrac{\sec^2 x}{2 - \sec^2 x}$

$$\frac{\left(\frac{1}{\cos^2 x}\right)\cos^2 x}{\left(2 - \frac{1}{\cos^2 x}\right)\cos^2 x}$$

$$\frac{1}{2\cos^2 x - 1}$$

$$\frac{1}{\cos 2x}$$

$$\sec 2x$$

(15) $\tan(\alpha + \beta)\tan(\alpha - \beta) = \dfrac{\tan^2\alpha - \tan^2\beta}{1 - \tan^2\alpha \tan^2\beta}$

$$\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} \cdot \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

$$\frac{\tan^2\alpha - \tan^2\beta}{1 - \tan^2\alpha\tan^2\beta}$$

(16) $\csc\theta + \cot\theta = \dfrac{\sin\theta}{1-\cos\theta}\dfrac{(1+\cos\theta)}{(1+\cos\theta)}$

$\dfrac{1}{\sin\theta} + \dfrac{\cos\theta}{\sin\theta}$

$\dfrac{1+\cos\theta}{\sin\theta}$

$\dfrac{\sin\theta + \sin\theta\cos\theta}{1-\cos^2\theta}$

$\dfrac{\sin\theta + \sin\theta\cos\theta}{\sin^2\theta}$

$\dfrac{\sin\theta\,(1+\cos\theta)}{\sin^2\theta}$

$\dfrac{1+\cos\theta}{\sin\theta}$

(17) $\dfrac{1}{\sec 2x} = \dfrac{2\tan x}{1+\tan^2 x}$

$\sin 2x = \dfrac{\left(2\cdot\dfrac{\sin x}{\cos x}\right)\cdot\cos^2 x}{\left(1+\dfrac{\sin^2 x}{\cos^2 x}\right)\cdot\cos^2 x}$

$\dfrac{2\sin x\cos x}{\cos^2 x + \sin^2 x}$

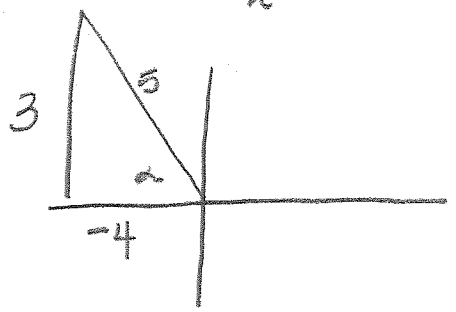
$\dfrac{2\sin x\cos x}{1}$

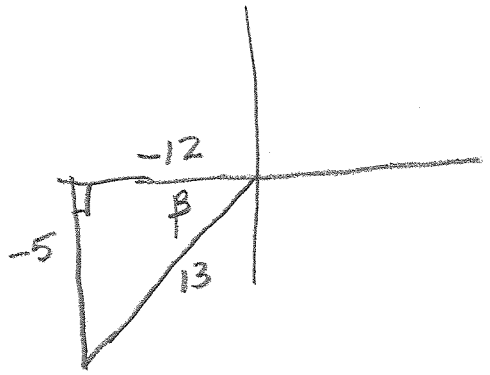
(18) $\dfrac{\sec x - 1}{x \sec x} = \dfrac{1 - \cos x}{x}$

$\dfrac{\sec x}{x \sec x} - \dfrac{1}{x \sec x}$

$\dfrac{1}{x} - \dfrac{1}{x} \cdot \cos x$

$\dfrac{1 - \cos x}{x}$





(19) $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\left(-\frac{4}{5}\right)\left(\frac{-12}{13}\right) + \left(\frac{3}{5}\right)\left(\frac{-5}{13}\right)$

$\frac{48}{65} - \frac{15}{65} = \boxed{\frac{33}{65}}$

(20) $\tan(\alpha + \beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$ $\qquad \dfrac{-\frac{3}{4} + \frac{-5}{12}}{1 - \left(-\frac{3}{4}\right)\left(\frac{5}{12}\right)}$ $\qquad \dfrac{-\frac{4}{12}}{1 + \frac{15}{48}}$

$= \dfrac{-\frac{1}{3}}{\frac{63}{48}} = -\frac{1}{3} \cdot \frac{48}{63}$

$\boxed{\frac{-16}{63}}$

(21) $\sin 2\alpha = 2\sin\alpha\cos\alpha$

$2\left(\frac{3}{5}\right)\left(-\frac{4}{5}\right) = -\frac{24}{25}$

(22) $\cos\frac{\beta}{2} = -\sqrt{\dfrac{1 + \cos\beta}{2}} \Rightarrow -\sqrt{\dfrac{1 + \left(-\frac{12}{13}\right)}{2}} = -\sqrt{\frac{1}{26}} = -\frac{\sqrt{26}}{26}$

$(\Omega \in \Pi)$

(23) $\sin\frac{3\pi}{4}\cos\frac{5\pi}{6} + \sin\frac{5\pi}{6}\cos\frac{3\pi}{4}$

$\frac{\sqrt{2}}{2}\cdot -\frac{\sqrt{3}}{2} + \frac{1}{2}\cdot -\frac{\sqrt{2}}{2} = -\frac{\sqrt{6}-\sqrt{2}}{4}$

(24) $\cos^2 15^\circ - \sin^2 15^\circ = \cos 2(15^\circ) = \cos 30^\circ = \frac{\sqrt{3}}{2}$

(25) $\cos\left(\frac{5\pi}{12} - \frac{\pi}{12}\right) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

(26) $\tan 22.5^\circ = \frac{1-\cos 45^\circ}{\sin 45^\circ}$ $\frac{1-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \frac{2-\sqrt{2}}{\sqrt{2}}$ $= \frac{2\sqrt{2}-2}{2}$ $\sqrt{2}-1$