where n is any integer.
The solutions in the interval $[0, 2\pi)$, are obtained by letting $n = 0$. The solutions are $\frac{\pi}{3}$ and $\frac{5\pi}{3}$.

Chapter 6 Review Exercises

1.
$$\begin{aligned}\sec x - \cos x &= \frac{1}{\cos x} - \cos x \\ &= \frac{1}{\cos x} - \frac{\cos x}{1} \cdot \frac{\cos x}{\cos x} \\ &= \frac{1}{\cos x} - \frac{\cos^2 x}{\cos x} \\ &= \frac{1 - \cos^2 x}{\cos x} \\ &= \frac{\sin^2 x}{\cos x} \\ &= \frac{\sin x}{\cos x} \cdot \sin x \\ &= \tan x \sin x\end{aligned}$$

2.
$$\begin{aligned}&\cos x + \sin x \tan x \\ &= \frac{\cos x}{\cos x} \cdot \cos x + \sin x \cdot \frac{\sin x}{\cos x} \\ &= \frac{\cos^2 x}{\cos x} + \frac{\sin^2 x}{\cos x} \\ &= \frac{\cos^2 x + \sin^2 x}{\cos x} \\ &= \frac{1}{\cos x} = \sec x\end{aligned}$$

3.
$$\begin{aligned}\sin^2\theta(1 + \cot^2\theta) &= \sin^2\theta + \sin^2\theta\cot^2\theta \\ &= \sin^2\theta + \sin^2\theta \cdot \frac{\cos^2\theta}{\sin^2\theta} \\ &= \sin^2\theta + \cos^2\theta \\ &= 1\end{aligned}$$

4.
$$\begin{aligned}(\sec\theta - 1)(\sec\theta + 1) &= \sec^2\theta - 1 \\ &= 1 + \tan^2\theta - 1 \\ &= \tan^2\theta\end{aligned}$$

5.
$$\begin{aligned}\frac{1 - \tan x}{\sin x} &= \frac{1}{\sin x} - \frac{\tan x}{\sin x} \\ &= \csc x - \frac{\sin x}{\cos x} \cdot \frac{1}{\sin x} \\ &= \csc x - \frac{1}{\cos x} \\ &= \csc x - \sec x\end{aligned}$$

6.
$$\begin{aligned}&\frac{1}{\sin t - 1} + \frac{1}{\sin t + 1} \\ &= \frac{1}{\sin t - 1} \cdot \frac{\sin t + 1}{\sin t + 1} + \frac{1}{\sin t + 1} \cdot \frac{\sin t - 1}{\sin t - 1} \\ &= \frac{\sin t + 1}{\sin^2 t - 1} + \frac{\sin t - 1}{\sin^2 t - 1} \\ &= \frac{\sin t + 1 + \sin t - 1}{\sin^2 t - 1} \\ &= \frac{2\sin t}{\sin^2 t - 1} \\ &= \frac{2\sin t}{-\cos^2 t} \\ &= -2 \cdot \frac{\sin t}{\cos t} \cdot \frac{1}{\cos t} \\ &= -2\tan t \sec t\end{aligned}$$

7.
$$\begin{aligned}\frac{1 + \sin t}{\cos^2 t} &= \frac{1}{\cos^2 t} + \frac{\sin t}{\cos^2 t} \\ &= \sec^2 t + \frac{\sin t}{\cos t} \cdot \frac{1}{\cos t} \\ &= \tan^2 t + 1 + \tan t \sec t\end{aligned}$$

8.
$$\begin{aligned}\frac{\cos x}{1 - \sin x} &= \frac{\cos x}{1 - \sin x} \cdot \frac{1 + \sin x}{1 + \sin x} \\ &= \frac{\cos x(1 + \sin x)}{1 - \sin^2 x} \\ &= \frac{\cos x(1 + \sin x)}{\cos^2 x} \\ &= \frac{1 + \sin x}{\cos x}\end{aligned}$$

9.
$$\begin{aligned}1 - \frac{\sin^2 x}{1 + \cos x} &= 1 - \frac{1 - \cos^2 x}{1 + \cos x} \\ &= 1 - \frac{(1 + \cos x)(1 - \cos x)}{1 + \cos x} \\ &= 1 - (1 - \cos x) \\ &= 1 - 1 + \cos x \\ &= \cos x\end{aligned}$$

10. $(\tan\theta+\cot\theta)^2$

$= \tan^2\theta+2\tan\theta\cot\theta+\cot^2\theta$

$= \sec^2\theta-1+2\dfrac{\sin\theta}{\cos\theta}\cdot\dfrac{\cos\theta}{\sin\theta}+\csc^2\theta-1$

$= \sec^2\theta-1+2+\csc^2\theta-1$

$= \sec^2\theta+\csc^2\theta$

11. $\dfrac{1}{\sin\theta+\cos\theta}+\dfrac{1}{\sin\theta-\cos\theta}$

$= \dfrac{\sin\theta-\cos\theta}{\sin\theta-\cos\theta}\cdot\dfrac{1}{\sin\theta+\cos\theta}+\dfrac{\sin\theta+\cos\theta}{\sin\theta+\cos\theta}\cdot\dfrac{1}{\sin\theta-\cos\theta}$

$= \dfrac{\sin\theta-\cos\theta}{\sin^2\theta-\cos^2\theta}+\dfrac{\sin\theta+\cos\theta}{\sin^2\theta-\cos^2\theta}$

$= \dfrac{2\sin\theta}{\sin^2\theta-\cos^2\theta}$

$= \dfrac{2\sin\theta}{\sin^2\theta-\cos^2\theta}\cdot\dfrac{\sin^2\theta+\cos^2\theta}{\sin^2\theta+\cos^2\theta}$

$= \dfrac{2\sin\theta\cdot 1}{\sin^4\theta-\cos^4\theta}$

$= \dfrac{2\sin\theta}{\sin^4\theta-\cos^4\theta}$

12. $\dfrac{\cos t}{\cot t-5\cos t}=\dfrac{\cos t}{\cot t-5\cos t}\cdot\dfrac{\frac{1}{\cos t}}{\frac{1}{\cos t}}$

$= \dfrac{\frac{\cos t}{\cos t}}{\frac{\cot t-5\cos t}{\cos t}}$

$= \dfrac{1}{\frac{\cot t}{\cos t}-5}$

$= \dfrac{1}{\frac{\frac{\cos t}{\sin t}}{\cos t}-5}$

$= \dfrac{1}{\frac{\cos t}{\sin t}\cdot\frac{1}{\cos t}-5}$

$= \dfrac{1}{\frac{1}{\sin t}-5}$

$= \dfrac{1}{\csc t-5}$

13. $\dfrac{1-\cos t}{1+\cos t}=\dfrac{1-\cos t}{1+\cos t}\cdot\dfrac{1-\cos t}{1-\cos t}$

$= \dfrac{(1-\cos t)^2}{1-\cos^2 t}$

$= \dfrac{(1-\cos t)^2}{\sin^2 t}$

$= \left(\dfrac{1-\cos t}{\sin t}\right)^2$

$= \left(\dfrac{1}{\sin t}-\dfrac{\cos t}{\sin t}\right)^2$

$= (\csc t-\cot t)^2$

14. $\cos(45°+30°)$

$= \cos 45°\cos 30°-\sin 45°\sin 30°$

$= \dfrac{\sqrt{2}}{2}\cdot\dfrac{\sqrt{3}}{2}-\dfrac{\sqrt{2}}{2}\cdot\dfrac{1}{2}$

$= \dfrac{\sqrt{6}}{4}-\dfrac{\sqrt{2}}{4}$

$= \dfrac{\sqrt{6}-\sqrt{2}}{4}$

15. $\sin 195° = \sin(135°+60°)$

$= \sin 135°\cos 60°+\cos 135°\sin 60°$

$= \dfrac{\sqrt{2}}{2}\cdot\dfrac{1}{2}+\left(-\dfrac{\sqrt{2}}{2}\right)\cdot\dfrac{\sqrt{3}}{2}$

$= \dfrac{\sqrt{2}}{4}-\dfrac{\sqrt{6}}{4}$

$= \dfrac{\sqrt{2}-\sqrt{6}}{4}$

16. $\tan\left(\dfrac{4\pi}{3}-\dfrac{\pi}{4}\right)=\dfrac{\tan\frac{4\pi}{3}-\tan\frac{\pi}{4}}{1+\tan\frac{4\pi}{3}\cdot\tan\frac{\pi}{4}}=\dfrac{\sqrt{3}-1}{1+\sqrt{3}\cdot(1)}$

$= \dfrac{(\sqrt{3}-1)(1-\sqrt{3})}{(1+\sqrt{3})(1-\sqrt{3})}=\dfrac{-(1-\sqrt{3})^2}{1-3}$

$\dfrac{-(1-2\sqrt{3}+3)}{-2}=\dfrac{1-2\sqrt{3}+3}{2}$

$= \dfrac{4-2\sqrt{3}}{2}=\dfrac{2(2-\sqrt{3})}{2}=2-\sqrt{3}$