Chapter 2 Review Exercises

function
domain: {2, 3, 5}
range: {7}

function
domain: {1, 2, 13}
range: {10, 500, π}

not a function
domain: {12, 14}
range: {13, 15, 19}

$2x+y=8$

$y=-2x+8$

Since only one value of y can be obtained for each value of x, y is a function of x.

$3x^2+y=14$

$y=-3x^2+14$

Since only one value of y can be obtained for each value of x, y is a function of x.

$2x+y^2=6$

$y^2=-2x+6$

$y=\pm\sqrt{-2x+6}$

Since more than one value of y can be obtained from some values of x, y is not a function of x.

$f(x) = 5 - 7x$

a. $f(4) = 5 - 7(4) = -23$

b. $f(x+3)=5-7(x+3)$
$=5-7x-21$
$=-7x-16$

c. $f(-x) = 5 - 7(-x) = 5 + 7x$

$g(x)=3x^2-5x+2$

a. $g(0)=3(0)^2-5(0)+2=2$

b. $g(-2)=3(-2)^2-5(-2)+2$
$=12+10+2$
$=24$

c. $g(x-1)=3(x-1)^2-5(x-1)+2$
$=3(x^2-2x+1)-5x+5+2$
$=3x^2-11x+10$

d. $g(-x)=3(-x)^2-5(-x)+2$
$=3x^2+5x+2$

9. a. $g(13)=\sqrt{13-4}=\sqrt{9}=3$

b. $g(0)=4-0=4$

c. $g(-3)=4-(-3)=7$

10. a. $f(-2)=\dfrac{(-2)^2-1}{-2-1}=\dfrac{3}{-3}=-1$

b. $f(1) = 12$

c. $f(2)=\dfrac{2^2-1}{2-1}=\dfrac{3}{1}=3$

11. The vertical line test shows that this is not the graph of a function.

12. The vertical line test shows that this is the graph of a function.

13. The vertical line test shows that this is the graph of a function.

14. The vertical line test shows that this is not the graph of a function.

15. The vertical line test shows that this is not the graph of a function.

16. The vertical line test shows that this is the graph of a function.

17. $\dfrac{8(x+h)-11-(8x-11)}{h}$
$=\dfrac{8x+8h-11-8x+11}{h}$
$=\dfrac{8h}{8}$
$=8$

18. $\dfrac{-2(x+h)^2+(x+h)+10-\left(-2x^2+x+10\right)}{h}$
$=\dfrac{-2\left(x^2+2xh+h^2\right)+x+h+10+2x^2-x-10}{h}$
$=\dfrac{-2x^2-4xh-2h^2+x+h+10+2x^2-x-10}{h}$
$=\dfrac{-4xh-2h^2+h}{h}$
$=\dfrac{h(-4x-2h+1)}{h}$
$-4x-2h+1$

19. **a.** domain: [–3, 5)

b. range: [–5, 0]

c. x-intercept: –3

d. y-intercept: –2

e. increasing: $(-2, 0)$ or $(3, 5)$
decreasing: $(-3, -2)$ or $(0, 3)$

f. $f(-2) = -3$ and $f(3) = -5$

20. **a.** domain: $(-\infty, \infty)$

b. range: $(-\infty, \infty)$

c. x-intercepts: –2 and 3

d. y-intercept: 3

e. increasing: (–5, 0)
decreasing: $(-\infty, -5)$ or $(0, \infty)$

f. $f(-2) = 0$ and $f(6) = -3$

21. **a.** domain: $(-\infty, \infty)$

b. range: [–2, 2]

c. x-intercept: 0

d. y-intercept: 0

e. increasing: (–2, 2)
constant: $(-\infty, -2)$ or $(2, \infty)$

f. $f(-9) = -2$ and $f(14) = 2$

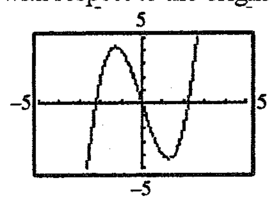
22. **a.** 0, relative maximum –2

b. –2, 3, relative minimum –3, –5

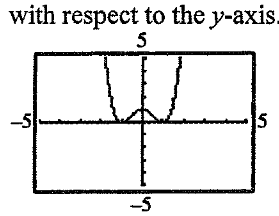
23. **a.** 0, relative maximum 3

b. –5, relative minimum –6

24. $f(x) = x^3 - 5x$
$f(-x) = (-x)^3 - 5(-x)$
$= -x^3 + 5x$
$= -f(x)$
The function is odd. The function is symmetric with respect to the origin.



25. $f(x) = x^4 - 2x^2 + 1$
$f(-x) = (-x)^4 - 2(-x)^2 + 1$
$= x^4 - 2x^2 + 1$
$= f(x)$
The function is even. The function is symmetric with respect to the y-axis.



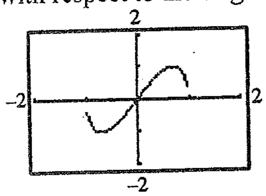
6. $f(x) = 2x\sqrt{1-x^2}$

$f(-x) = 2(-x)\sqrt{1-(-x)^2}$

$= -2x\sqrt{1-x^2}$

$= -f(x)$

The function is odd. The function is symmetric with respect to the origin.



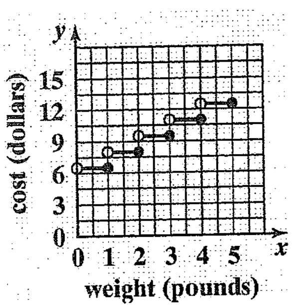
27. a. Yes, the eagle's height is a function of time since the graph passes the vertical line test.

b. Decreasing: (3, 12)
The eagle descended.

c. Constant: (0, 3) or (12, 17)
The eagle's height held steady during the first 3 seconds and the eagle was on the ground for 5 seconds.

d. Increasing: (17, 30)
The eagle was ascending.

28.



29. $m = \frac{1-2}{5-3} = \frac{-1}{2} = -\frac{1}{2}$; falls

30. $m = \frac{-4-(-2)}{-3-(-1)} = \frac{-2}{-2} = 1$; rises

31. $m = \frac{\frac{1}{4}-\frac{1}{4}}{6-(-3)} = \frac{0}{9} = 0$; horizontal

32. $m = \frac{10-5}{-2-(-2)} = \frac{5}{0}$ undefined; vertical

33. point-slope form: $y - 2 = -6(x + 3)$
slope-intercept form: $y = -6x - 16$

34. $m = \frac{2-6}{-1-1} = \frac{-4}{-2} = 2$
point-slope form: $y - 6 = 2(x - 1)$
or $y - 2 = 2(x + 1)$
slope-intercept form: $y = 2x + 4$

35. $3x + y - 9 = 0$
$y = -3x + 9$
$m = -3$
point-slope form:
$y + 7 = -3(x - 4)$
slope-intercept form:
$y = -3x + 12 - 7$
$y = -3x + 5$

36. perpendicular to $y = \frac{1}{3}x + 4$
$m = -3$
point-slope form:
$y - 6 = -3(x + 3)$
slope-intercept form:
$y = -3x - 9 + 6$
$y = -3x - 3$

37. Write $6x - y - 4 = 0$ in slope intercept form.

$6x - y - 4 = 0$

$-y = -6x + 4$

$y = 6x - 4$

The slope of the perpendicular line is 6, thus the slope of the desired line is $m = -\frac{1}{6}$.

$y - y_1 = m(x - x_1)$

$y - (-1) = -\frac{1}{6}(x - (-12))$

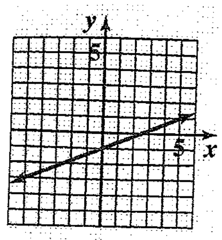
$y + 1 = -\frac{1}{6}(x + 12)$

$y + 1 = -\frac{1}{6}x - 2$

$6y + 6 = -x - 12$

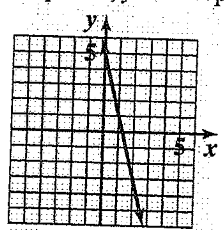
$x + 6y + 18 = 0$

38. slope: $\frac{2}{5}$; y-intercept: -1



$y = \frac{2}{5}x - 1$

39. slope: -4; y-intercept: 5



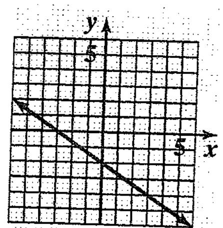
$f(x) = -4x + 5$

40. $2x + 3y + 6 = 0$

$3y = -2x - 6$

$y = -\frac{2}{3}x - 2$

slope: $-\frac{2}{3}$; y-intercept: -2



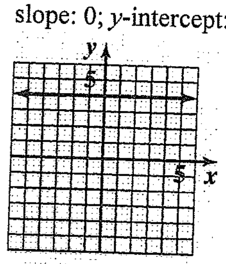
$2x + 3y + 6 = 0$

41. $2y - 8 = 0$

$2y = 8$

$y = 4$

slope: 0; y-intercept: 4



$2y - 8 = 0$

42. $2x - 5y - 10 = 0$

Find x-intercept:

$2x - 5(0) - 10 = 0$

$2x - 10 = 0$

$2x = 10$

$x = 5$

Find y-intercept:

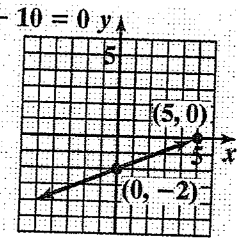
$2(0) - 5y - 10 = 0$

$-5y - 10 = 0$

$-5y = 10$

$y = -2$

$2x - 5y - 10 = 0$

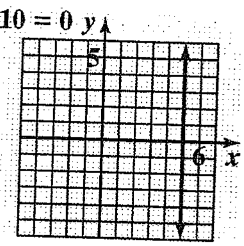


43. $2x - 10 = 0$

$2x = 10$

$x = 5$

$2x - 10 = 0$



44. a. First, find the slope. $(1, 1.5)$ and $(3, 3.4)$.

$$m = \frac{3.4 - 1.5}{3 - 1} = \frac{1.9}{2} = 0.95$$

Next, use the slope and one of the points to write the point-slope equation of the line.

$y - 1.5 = 0.95(x - 1)$ or $y - 3.4 = 0.95(x - 3)$

b. $y - 1.5 = 0.95(x - 1)$

$y - 1.5 = 0.95x - 0.95$

$y = 0.95x - 0.55$

c. Since 2009 is 2009-1999 = 10, let $x = 10$.

$y = 0.95(10) + 0.55$

$= 9.5 + 0.55 = 10.05$

$10.05 billion in revenue was earned from online gambling in 2009.

45. **a.** $(1999, 41315)$ and $(2001, 41227)$

$$m = \frac{41227 - 41315}{2001 - 1999} = \frac{-88}{2} = -44$$

The number of new AIDS diagnoses decreased at a rate of 44 each year from 1999 to 2001.

b. $(2001, 41227)$ and $(2003, 43045)$

$$m = \frac{43045 - 41227}{2003 - 2001} = \frac{1818}{2} = 909$$

The number of new AIDS diagnoses increased at a rate of 909 each year from 2001 to 2003.

c. $(1999, 41315)$ and $(2003, 43045)$

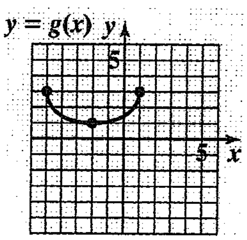
$$m = \frac{43045 - 41315}{2003 - 1999} = \frac{1730}{4} = 432.5$$

$$\frac{-44 + 909}{2} = \frac{865}{2} = 432.5$$

Yes, the slope equals the average of the two values.

46. $$\frac{9^2 - 4(9) - [4^2 - 4 \cdot 5]}{9 - 5} = \frac{40}{4} = 10$$

47.



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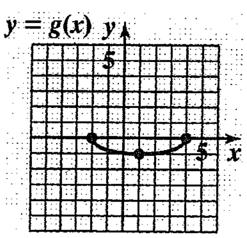

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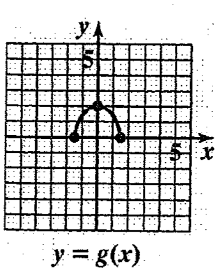

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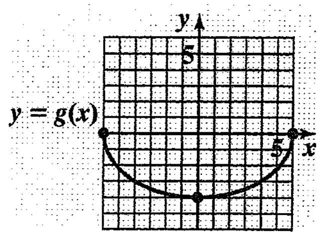

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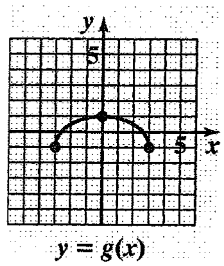

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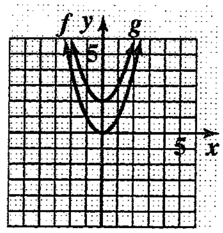

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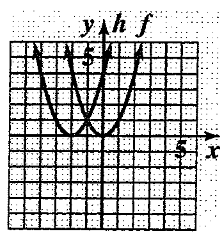

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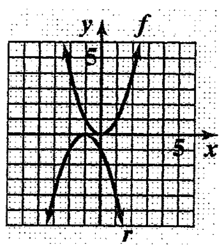

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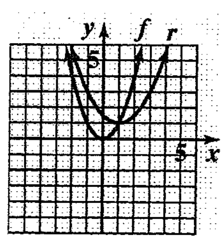

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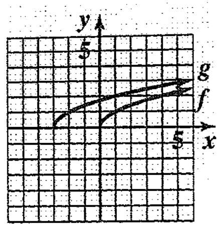

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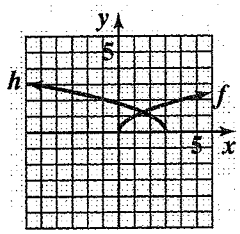

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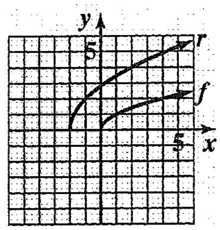

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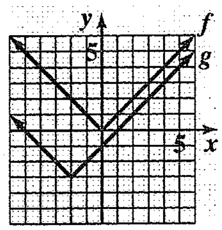

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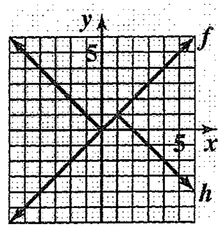

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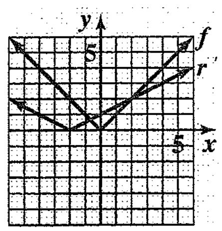

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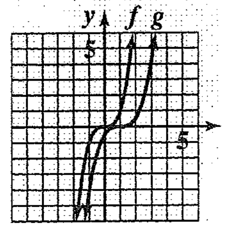

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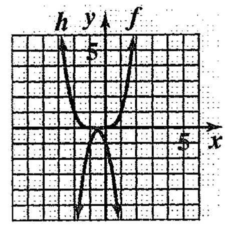

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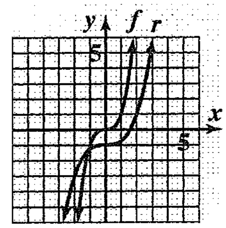

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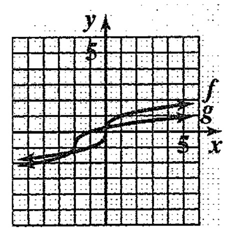

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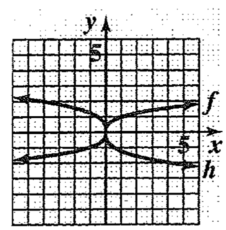

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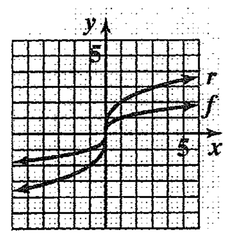


68. domain: $(-\infty, \infty)$

69. The denominator is zero when $x = 7$. The domain is $(-\infty, 7)\cup(7, \infty)$.

70. The expressions under each radical must not be negative.
$8 - 2x \ge 0$
$-2x \ge -8$
$x \le 4$
Domain: $(-\infty, 4]$.

71. The denominator is zero when $x = -7$ or $x = 3$.
Domain: $(-\infty,-7)\cup(-7,3)\cup(3,\infty)$

72. The expressions under each radical must not be negative. The denominator is zero when $x = 5$.
$x - 2 \ge 0$
$x \ge 2$
Domain: $[2,5)\cup(5,\infty)$

73. The expressions under each radical must not be negative.
$x-1\ge 0$ and $x+5\ge 0$
$x \ge 1$ $\quad x \ge -5$
Domain: $[1,\infty)$

74. $f(x) = 3x - 1$; $g(x) = x - 5$
$(f+g)(x) = 4x - 6$
Domain: $(-\infty, \infty)$
$(f-g)(x) = (3x-1)-(x-5) = 2x+4$
Domain: $(-\infty, \infty)$
$(fg)(x) = (3x-1)(x-5) = 3x^2 - 16x + 5$
Domain: $(-\infty, \infty)$
$\left(\frac{f}{g}\right)(x) = \frac{3x-1}{x-5}$
Domain: $(-\infty,5)\cup(5,\infty)$

75. $f(x) = x^2 + x + 1$; $g(x) = x^2 - 1$
$(f+g)(x) = 2x^2 + x$
Domain: $(-\infty, \infty)$
$(f-g)(x) = (x^2+x+1)-(x^2-1) = x+2$
Domain: $(-\infty, \infty)$
$(fg)(x) = (x^2+x+1)(x^2-1)$
$= x^4 + x^3 - x - 1$
$\left(\frac{f}{g}\right)(x) = \frac{x^2+x+1}{x^2-1}$
Domain: $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$

76. $f(x) = \sqrt{x+7}$; $g(x) = \sqrt{x-2}$
$(f+g)(x) = \sqrt{x+7} + \sqrt{x-2}$
Domain: $[2, \infty)$
$(f-g)(x) = \sqrt{x+7} - \sqrt{x-2}$
Domain: $[2, \infty)$
$(fg)(x) = \sqrt{x+7}\cdot\sqrt{x-2}$
$= \sqrt{x^2+5x-14}$
Domain: $[2, \infty)$
$\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x+7}}{\sqrt{x-2}}$
Domain: $(2, \infty)$

77. $f(x) = x^2 + 3$; $g(x) = 4x - 1$

a. $(f \circ g)(x) = (4x-1)^2 + 3$
$= 16x^2 - 8x + 4$

b. $(g \circ f)(x) = 4(x^2+3) - 1$
$= 4x^2 + 11$

c. $(f \circ g)(3) = 16(3)^2 - 8(3) + 4 = 124$

78. $f(x) = \sqrt{x}$; $g(x) = x + 1$

a. $(f \circ g)(x) = \sqrt{x+1}$

b. $(g \circ f)(x) = \sqrt{x} + 1$

c. $(f \circ g)(3) = \sqrt{3+1} = \sqrt{4} = 2$

79. **a.** $(f \circ g)(x) = f\left(\frac{1}{x}\right)$

$$= \frac{\frac{1}{x}+1}{\frac{1}{x}-2} = \frac{\left(\frac{1}{x}+1\right)x}{\left(\frac{1}{x}-2\right)x} = \frac{1+x}{1-2x}$$

b. $x \ne 0$ $\quad 1-2x \ne 0$
$x \ne \frac{1}{2}$
$(-\infty,0)\cup\left(0,\frac{1}{2}\right)\cup\left(\frac{1}{2},\infty\right)$

80. a. $(f \circ g)(x) = f(x+3) = \sqrt{x+3-1} = \sqrt{x+2}$

b. $x+2 \geq 0$
$x \geq -2$ $\quad [-2, \infty)$

81. $f(x) = x^4 \qquad g(x) = x^2 + 2x - 1$

82. $f(x) = \sqrt[3]{x} \qquad g(x) = 7x + 4$

83. $f(x) = \frac{3}{5}x + \frac{1}{2};\ g(x) = \frac{5}{3}x - 2$

$$f(g(x)) = \frac{3}{5}\left(\frac{5}{3}x - 2\right) + \frac{1}{2} = x - \frac{6}{5} + \frac{1}{2} = x - \frac{7}{10}$$

$$g(f(x)) = \frac{5}{3}\left(\frac{3}{5}x + \frac{1}{2}\right) - 2 = x + \frac{5}{6} - 2 = x - \frac{7}{6}$$

f and g are not inverses of each other.

84. $f(x) = 2 - 5x;\ g(x) = \frac{2-x}{5}$

$$f(g(x)) = 2 - 5\left(\frac{2-x}{5}\right) = 2 - (2-x) = x$$

$$g(f(x)) = \frac{2-(2-5x)}{5} = \frac{5x}{5} = x$$

f and g are inverses of each other.

85. a.
$$f(x) = 4x - 3$$
$$y = 4x - 3$$
$$x = 4y - 3$$
$$y = \frac{x+3}{4}$$
$$f^{-1}(x) = \frac{x+3}{4}$$

b.
$$f(f^{-1}(x)) = 4\left(\frac{x+3}{4}\right) - 3 = x + 3 - 3 = x$$

$$f^{-1}(f(x)) = \frac{(4x-3)+3}{4} = \frac{4x}{4} = x$$

86. a.
$$f(x) = 8x^3 + 1$$
$$y = 8x^3 + 1$$
$$x = 8y^3 + 1$$
$$x - 1 = 8y^3$$
$$\frac{x-1}{8} = y^3$$
$$\sqrt[3]{\frac{x-1}{8}} = y$$
$$\frac{\sqrt[3]{x-1}}{2} = y$$
$$f^{-1}(x) = \frac{\sqrt[3]{x-1}}{2}$$

b.
$$f\left(f^{-1}(x)\right) = 8\left(\frac{\sqrt[3]{x-1}}{2}\right)^3 + 1 = 8\left(\frac{x-1}{8}\right) + 1 = x - 1 + 1 = x$$

$$f^{-1}\left(f(x)\right) = \frac{\sqrt[3]{\left(8x^3+1\right)-1}}{2} = \frac{\sqrt[3]{8x^3}}{2} = \frac{2x}{2} = x$$

. a. $f(x)=\frac{2}{x}+5$

$y=\frac{2}{x}+5$

$x=\frac{2}{y}+5$

$xy=2+5y$

$xy-5y=2$

$y(x-5)=2$

$y=\frac{2}{x-5}$

$f^{-1}(x)=\frac{2}{x-5}$

b. $f\left(f^{-1}(x)\right)=\frac{2}{\frac{2}{x-5}}+5$

$=\frac{2(x-5)}{2}+5$

$=x-5+5$

$=x$

$f^{-1}\left(f(x)\right)=\frac{2}{\frac{2}{x}+5-5}$

$=\frac{2}{\frac{2}{x}}$

$=\frac{2x}{2}$

$=x$

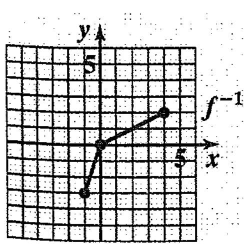
8. The inverse function exists.

9. The inverse function does not exist since it does not pass the horizontal line test.

0. The inverse function exists.

1. The inverse function does not exist since it does not pass the horizontal line test.

2.



93. $f(x)=1-x^2$

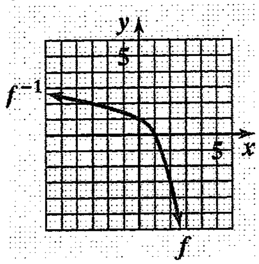
$y=1-x^2$

$x=1-y^2$

$y^2=1-x$

$y=\sqrt{1-x}$

$f^{-1}(x)=\sqrt{1-x}$



94. $f(x)=\sqrt{x}+1$

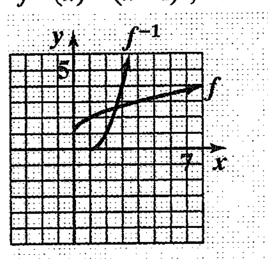
$y=\sqrt{x}+1$

$x=\sqrt{y}+1$

$x-1=\sqrt{y}$

$(x-1)^2=y$

$f^{-1}(x)=(x-1)^2,\quad x\geq 1$



$f(x)=\sqrt{x}+1$

$g(x)=(x-1)^2, x\geq 1$

95. $d=\sqrt{[3-(-2)]^2+[9-(-3)]^2}$

$=\sqrt{5^2+12^2}$

$=\sqrt{25+144}$

$=\sqrt{169}$

$=13$

96. $d = \sqrt{[-2-(-4)]^2 + (5-3)^2}$
$= \sqrt{2^2 + 2^2}$
$= \sqrt{4+4}$
$= \sqrt{8}$
$= 2\sqrt{2}$
≈ 2.83

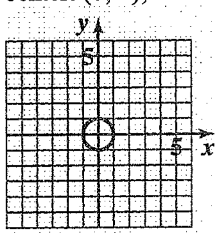
97. $\left(\frac{2+(-12)}{2}, \frac{6+4}{2}\right) = \left(\frac{-10}{2}, \frac{10}{2}\right) = (-5, 5)$

98. $\left(\frac{4+(-15)}{2}, \frac{-6+2}{2}\right) = \left(\frac{-11}{2}, \frac{-4}{2}\right) = \left(\frac{-11}{2}, -2\right)$

99. $x^2 + y^2 = 3^2$
$x^2 + y^2 = 9$

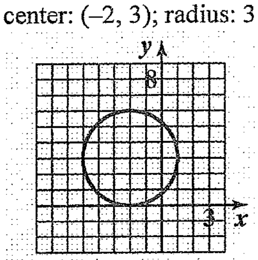
100. $(x-(-2))^2 + (y-4)^2 = 6^2$
$(x+2)^2 + (y-4)^2 = 36$

101. center: (0, 0); radius: 1



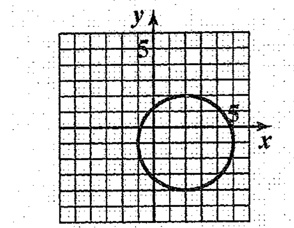
$x^2 + y^2 = 1$
Domain: $[-1, 1]$
Range: $[-1, 1]$

102. center: (–2, 3); radius: 3



$(x+2)^2 + (y-3)^2 = 9$
Domain: $[-5, 1]$
Range: $[0, 6]$

103. $x^2 + y^2 - 4x + 2y - 4 = 0$
$x^2 - 4x \quad + y^2 + 2y \quad = 4$
$x^2 - 4x + 4 + y^2 + 2y + 1 = 4 + 4 + 1$
$(x-2)^2 + (y+1)^2 = 9$
center: (2, –1); radius: 3



$x^2 + y^2 - 4x + 2y - 4 = 0$
Domain: $[-1, 5]$
Range: $[-4, 2]$

Chapter 2 Test

1. (b), (c), and (d) are not functions.

2. **a.** $f(4) - f(-3) = 3 - (-2) = 5$

b. domain: (–5, 6]

c. range: [–4, 5]

d. increasing: (–1, 2)

e. decreasing: (–5, –1) or (2, 6)

f. $2, f(2) = 5$

g. (–1, –4)

h. x-intercepts: –4, 1, and 5.

i. y-intercept: –3

3. **a.** –2, 2

b. –1, 1

c. 0

d. even; $f(-x) = f(x)$

e. no; f fails the horizontal line test

f. $f(0)$ is a relative minimum.