

③

$$\mathbb{P}_g = \pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{3}, \pm \frac{2}{3}$$

⑥

$$\mathbb{P}_g = \pm 1, \pm 2, \pm 4, \pm 8, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm \frac{8}{3}$$

also factors

$$x^2(x+1) - 4(x+1) = (x+1)(x+2)(x-2)$$

$$(9) f(x) = x^3 + x^2 - 4x - 4$$

$$\frac{p}{q} = \pm 1, \pm 2, \pm 4$$

$$\begin{array}{r|rrrrr} 2 & 1 & 1 & -4 & -4 & \\ & & 2 & 6 & 4 & \\ \hline & 1 & 3 & 2 & 0 & \end{array}$$

$$x^2 + 3x + 2 = 0$$

$$(x+2)(x+1) = 0$$

$$\{-1, -2, 2\}$$

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$$f(x) = 2x^2 - 5x + x + 2$$

$$\frac{p}{q} = \pm 1, \pm 2, \pm \frac{1}{2}$$

$$\begin{array}{r|rrrr} 2 & 2 & -5 & 1 & 2 \\ & & 4 & -2 & -2 \\ \hline & 2 & -1 & -1 & 0 \end{array}$$

$$2x^2 - x - 1$$
$$(2x + 1)(x - 1) = 0$$

$$\left\{ -\frac{1}{2}, 1, 2 \right\}$$

15)

$$p \pm q = \pm 1 \pm 2 \pm \frac{1}{2}$$

$$\begin{array}{r} -2 \overline{) \quad 2 \quad 6 \quad 5 \quad 2} \\ \underline{\quad \quad -4 \quad -4 \quad -2} \\ \quad \quad 2 \quad 2 \quad 1 \quad 0 \end{array}$$

$$2x^2 + 2x + 1 = 0$$

$$\frac{-2 \pm \sqrt{4-8}}{4}$$

$$\frac{-2 \pm 2i}{4} = \frac{-1 \pm i}{2}$$

$$\left\{ -2, \frac{-1 \pm i}{2} \right\}$$

$$(18) \frac{p}{q} = \pm 1, \pm 2, \pm 4$$

$$\{4, -1\}$$

M2.

$$\begin{array}{r} 4 \overline{) 1 \quad -2 \quad -7 \quad -4} \\ \underline{ 4 \quad 8 \quad 4} \\ 1 \quad 2 \quad 1 \quad 0 \end{array}$$

$$x^2 + 2x + 1 = 0$$
$$(x+1)^2$$

(21)

$$g = \pm 1, \pm 5, \pm \frac{1}{2}, \pm \frac{5}{2}, \pm \frac{1}{3}$$
$$\pm \frac{5}{3}, \pm \frac{1}{6}, \pm \frac{5}{6}$$

$$\begin{array}{r} -5 \overline{) 6 \quad 25 \quad -24 \quad 5} \\ \underline{ 30} \\ -30 \\ \underline{ 25} \\ -25 \\ \underline{ 0} \\ -5 \end{array}$$

$$\begin{array}{r} 6 \quad -5 \quad 1 \\ \hline 6x^2 - 5x + 1 = 0 \\ (2x-1)(3x-1) = 0 \end{array} \quad \left\{ -5, \frac{1}{2}, \frac{1}{3} \right\}$$

$$(24) \quad P_9 = \pm 1, \pm 3, \pm 5, \pm 15$$

$$\begin{array}{r|rrrrrr} 3 & 1 & 0 & -2 & -16 & -15 \\ & & 3 & 9 & 21 & 15 \\ \hline -1 & 1 & 3 & 7 & 5 & 0 \\ & & -1 & -2 & -5 & \end{array}$$

$$x^2 + 2x + 5 = 0$$

$$\{3, -1, -1 \pm 2i\}$$

$$\frac{-2 \pm \sqrt{4 - 20}}{2}$$

$$\frac{-2 \pm 4i}{2} = -1 \pm 2i$$

$$(27) (x+5)(x-(4-3i))(x-(4+3i))=0$$

$$(x+5)(x^2 - x(4-3i) - x(4+3i) + (4-3i)(4+3i))$$

$$(x+5)(x^2 - 4x + 3ix - 4x - 3ix + 25)$$

$$(x+5)(x^2 - 8x + 25) = 0$$

$$x^3 - 8x^2 + 25x + 5x^2 - 40x + 125 = 0$$

$$x^3 - 3x^2 - 15x + 125 = 0$$

$$f(x) = a_n(x^3 - 3x^2 - 15x + 125)$$

when $f(x) = 91$ $x = 2$ from table \rightarrow calc

$$91 = a_n(2^3 - 3(2)^2 - 15(2) + 125)$$

so $91 = a_n(91)$

$$a_n = 1 \quad f(x) = x^3 - 3x^2 - 15x + 125$$

$$\textcircled{30} \quad (x+2)\left(x+\frac{1}{2}\right)(x-i)(x+i)=0$$

$$(x+2)(2x+1)(x-i)(x+i)=0$$

$$(2x^2+5x+2)(x^2-xi+xi-i^2)$$

$$(2x^2+5x+2)(x^2+1)=0$$

$$2x^4+2x^2+5x^3+5x+2x^2+2=0$$

$$2x^4+5x^3+4x^2+5x+2=0$$

$$f(x)=18 \text{ at } x=1$$

$$18 = a_n (2(1)^4 + 5(1)^3 + 4(1)^2 + 5(1) + 2)$$

$$18 = a_n (18)$$

$$a_n = 1$$

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$$P_g = \pm 1, \pm 2, \pm 5, \pm 10$$

$$\begin{array}{r|rrrr} -2 & 1 & -4 & -7 & 10 \\ & & -2 & 12 & -10 \\ \hline & 1 & -6 & 5 & 0 \end{array}$$

$$x^2 - 6x + 5 = 0$$

$$(x - 5)(x - 1)$$

{ 1, 5, -2 }

(42)

$$p = \pm 1, \pm 2, \pm 4, \pm 8, \pm \frac{1}{3}, \pm \frac{2}{3}$$
$$q = \pm \frac{4}{3}, \pm \frac{8}{3}$$

$$\begin{array}{r} \sqrt{\frac{10}{3}} \quad | \quad 3 \quad -8 \quad -8 \quad 8 \\ \phantom{\sqrt{\frac{10}{3}} \quad |} \\ \phantom{\sqrt{\frac{10}{3}} \quad |} \\ \phantom{\sqrt{\frac{10}{3}} \quad |} \\ \phantom{\sqrt{\frac{10}{3}} \quad |} \\ \hline \phantom{\sqrt{\frac{10}{3}} \quad |} 3 \quad -6 \quad -12 \quad 0 \end{array}$$

$$3x^2 - 6x - 12 = 0$$

$\left\{ \frac{10}{3}, 1 \pm \sqrt{5} \right\}$

$$x^2 - 2x - 4 = 0$$

$$x = \frac{2 \pm \sqrt{4+16}}{2} = \frac{2 \pm 2\sqrt{5}}{2}$$
$$= 1 \pm \sqrt{5}$$

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$$\frac{p}{q} = \pm 1, \pm 2, \pm 4, \pm 8$$

$$\begin{array}{r|rrrrrr} -1 & 1 & -3 & -20 & -24 & -8 \\ & & & -1 & 4 & 16 & 8 \end{array}$$

$$\begin{array}{r|rrrrr} -2 & 1 & -4 & -16 & -8 & 0 \\ & & -2 & 12 & 8 & \end{array}$$

$$\begin{array}{r|rrrr} & 1 & -6 & -4 & 0 \end{array}$$

$$x^2 - 6x - 4 = 0$$

$$\left\{ -1, -2, 3 \pm \sqrt{13} \right\} \begin{cases} x = \frac{6 \pm \sqrt{36+16}}{2} \\ x = \frac{6 \pm 2\sqrt{13}}{2} = 3 \pm \sqrt{13} \end{cases}$$

(48) $\pm 1, \pm 3, \pm 5, \pm 15, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{5}{2}, \pm \frac{15}{2}$

$$\begin{array}{r}
 1 \mid 2 \quad 3 \quad -11 \quad -9 \quad 15 \\
 \quad \quad \quad 2 \quad 5 \quad -6 \quad -15 \\
 \hline
 -\frac{5}{2} \mid 2 \quad 5 \quad -6 \quad -15 \quad 0 \\
 \quad \quad \quad -5 \quad 0 \quad 15 \\
 \hline
 \quad \quad 2 \quad 0 \quad -6 \quad 0
 \end{array}$$

$$2x^2 - 6 = 0$$

$$2x^2 = 6$$

$$x^2 = 3$$

$$x = \pm \sqrt{3}$$

$$\left\{ 1, \frac{5}{2}, \pm \sqrt{3} \right\}$$

$$(51) \frac{p}{q} = \pm 1, \pm 2, \pm 4, \pm 8, \pm \frac{1}{2}$$

$$\begin{array}{r|rrrrrr} -2 & 2 & 7 & 0 & -18 & -8 & 8 \\ & & -4 & -6 & 12 & 12 & -8 \\ \hline \end{array}$$

$$\begin{array}{r|rrrrrr} \frac{1}{2} & 2 & 3 & -6 & -6 & 4 & 0 \\ & & 1 & 2 & -2 & -4 & \\ \hline \end{array}$$

$$\begin{array}{r|rrrr} -2 & 2 & 4 & -4 & -8 \\ & & -4 & 0 & 8 \\ \hline & 2 & 0 & -4 & 10 \\ \hline \end{array}$$

$$2x^2 - 4 = 0$$

$$2x^2 = 4$$

$$x^2 = 2$$

$$\left\{ \frac{1}{2}, -2, \pm\sqrt{2} \right\}_{M_2} x = \pm\sqrt{2}$$

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$$\begin{array}{r} \underbrace{-1)}{-1 \quad 3 \quad 0 \quad -4} \\ \underline{ } \\ -1 \quad 4 \quad -4 \quad 0 \end{array}$$

$$-x^2 + 4x - 4 = 0$$

$$x^2 - 4x + 4 = 0$$

$$(x - 2)^2$$

$$\left\{ -1, 2 \right\} N_2$$

(57)

$$\left[0, 1, \frac{1}{4}\right]$$

$$\begin{array}{r|rrrrr} \frac{1}{2} & 2 & -3 & -7 & -8 & 6 \\ & & 1 & -1 & -4 & -6 \\ \hline 3 & 2 & -2 & -8 & -12 & 0 \\ & & 6 & 12 & 12 & \\ \hline & 2 & 4 & 4 & 0 & \end{array}$$

$$2x^2 + 4x + 4 = 6$$

$$\left\{ \frac{1}{2}, 3 - 1 \pm i \right\} \quad x^2 + 2x + 2 = 0$$

$$x = \frac{-2 \pm \sqrt{4 - 8}}{2}$$

$$x = \frac{-2 \pm 2i}{2} = -1 \pm i$$

(60)

$$\begin{array}{r|rrrrr} 1 & -5 & 4 & -19 & 16 & 4 \\ & & -5 & -1 & -20 & -4 \\ \hline -\frac{1}{5} & -5 & -1 & -20 & -4 & 0 \\ & & 1 & 0 & 4 & \\ \hline & -5 & 0 & -20 & 0 & \end{array}$$

$$\left\{ 1, -\frac{1}{5}, \pm 2i \right\}$$

$$-5x^2 - 20 = 0$$

$$5x^2 = -20$$

$$x^2 = -4$$

$$x = \pm 2i$$

$$(67) \quad x(10-2x)(8-2x) = 48$$

$$x(80 - 36x + 4x^2) = 48$$

$$80x - 36x^2 + 4x^3 = 48$$

$$4x^3 - 36x^2 + 80x - 48 = 0$$

$$x^3 - 9x^2 + 20x - 12 = 0$$

$$\begin{array}{r|rrrr} 2 & 1 & -9 & 20 & -12 \\ & & 2 & -14 & 12 \\ \hline & 1 & -7 & 6 & 0 \end{array}$$

$$x^2 - 7x + 6 = 0$$

$$(x-6)(x-1) = 0$$

but check.

$$6(10 - 2(6))(8 - 2(6))$$

$$= \checkmark$$

6 is extraneous

lin
or 2 in

89

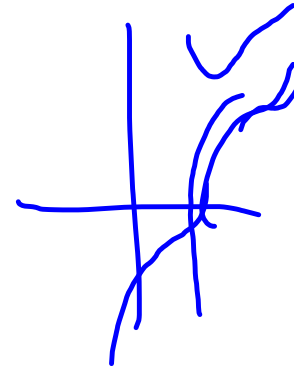
$$(2x+1)(x+5)(x+2) - 3x(x+5) = 208$$

$$(2x+1)(x^2+7x+10) - 3x^2 - 15x = 208$$

$$2x^3 + 14x^2 + 20x + x^2 + 7x + 10 - 3x^2 - 15x = 208$$

$$2x^3 + 12x^2 + 12x - 198 = 0$$

$$x^3 + 6x^2 + 6x - 99 = 0$$



$$\begin{array}{r|rrrr} 3 & 1 & 6 & 6 & -99 \\ & & 3 & 27 & 99 \\ \hline & 1 & 9 & 33 & 0 \end{array}$$

$$x = 3$$

$x^2 + 9x + 33 \rightarrow$ imaginary roots ✓