7. $\dfrac{\sin x}{1+\cos x}+\dfrac{1+\cos}{\sin x}$

$$=\frac{\sin x(\sin x)}{(1+\cos x)\sin x}+\frac{(1+\cos x)(1+\cos x)}{\sin x(1+\cos x)}$$
$$=\frac{\sin^2 x}{(1+\cos x)\sin x}+\frac{1+2\cos x+\cos^2 x}{(1+\cos x)\sin x}$$
$$=\frac{\sin^2 x+\cos^2 x+2\cos x+1}{(1+\cos x)\sin x}$$
$$=\frac{1+1+2\cos x}{(1+\cos x)\sin x}$$
$$=\frac{2+2+\cos x}{(1+\cos x)\sin x}$$
$$=\frac{2(1+\cos x)}{(1+\cos x)\sin x}$$
$$=\frac{2}{\sin x}$$
$$=2\csc x$$

We worked with the left side and arrived at the right side. Thus, the identity is verified.

8. Left side:

$$\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}$$
$$=\frac{1(1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}+\frac{1(1+\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}$$
$$=\frac{1-\sin\theta+1+\sin\theta}{(1+\sin\theta)(1-\sin\theta)}$$
$$=\frac{2}{(1+\sin\theta)(1-\sin\theta)}$$
$$=\frac{2}{1-\sin^2\theta}$$

Right side:

$$2+2\tan^2\theta=2+2\left(\frac{\sin^2\theta}{\cos^2\theta}\right)$$
$$=\frac{2\cos^2\theta}{\cos^2\theta}+\frac{2\sin^2\theta}{\cos^2\theta}$$
$$=\frac{2\cos^2\theta+2\sin^2\theta}{\cos^2\theta}$$
$$=\frac{2}{\cos^2\theta}=\frac{2}{1-\sin^2\theta}$$

The identity is verified because both sides are equal to $\dfrac{2}{1-\sin^2\theta}$.

Exercise Set 6.1

1. $\sin x\sec x=\sin x\cdot\dfrac{1}{\cos x}$

$$=\frac{\sin x}{\cos x}$$
$$=\tan x$$

2. $2\cos x\csc x=\cos x\cdot\dfrac{1}{\sin x}$

$$=\frac{\cos x}{\sin x}$$
$$=\cot x$$

3. $\tan(-x)\cdot\cos x=-\tan x\cdot\cos x$

$$=-\frac{\sin x}{\cos x}\cdot\cos x$$
$$=-\sin x$$

4. $\cot(-x)\sin x=-\cot x\sin x$

$$=-\frac{\cos x}{\sin x}\cdot\sin x$$
$$=-\cos x$$

5. $\tan x\csc x\cos x=\dfrac{\sin x}{\cos x}\cdot\dfrac{1}{\sin x}\cos x$

$$=1$$

6. $\cot x\sec x\sin x=\dfrac{\cos x}{\sin x}\cdot\dfrac{1}{\cos x}\cdot\sin x$

$$=1$$

7. $\sec x-\sec x\sin^2 x=\sec x(1-\sin^2 x)$

$$=\frac{1}{\cos x}\cdot\cos^2 x$$
$$=\cos x$$

8. $\csc x-\csc x\cos^2 x=\csc x(1-\cos^2 x)$

$$=\frac{1}{\sin x}\cdot\sin x^2$$
$$=\sin x$$

9. $\cos^2 x-\sin^2 x=\left(1-\sin^2 x\right)-\sin^2 x$

$$=1-\sin^2 x-\sin^2 x$$
$$=1-2\sin^2 x$$

10. $\cos^2 x - \sin^2 x = \cos^2 x - (1 - \cos^2 x)$
$= \cos^2 x - 1 + \cos^2 x$
$= 2\cos^2 x - 1$

11. $\csc\theta - \sin\theta = \dfrac{1}{\sin\theta} - \sin\theta$
$= \dfrac{1}{\sin\theta} - \dfrac{\sin^2\theta}{\sin\theta}$
$= \dfrac{1-\sin^2\theta}{\sin\theta}$
$= \dfrac{\cos^2\theta}{\sin\theta}$
$= \dfrac{\cos\theta}{\sin\theta}\cdot\cos\theta$
$= \cot\theta\cos\theta$

12. $\tan\theta + \cot\theta = \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$
$= \dfrac{\sin\theta}{\cos\theta}\cdot\dfrac{\sin\theta}{\sin\theta} + \dfrac{\cos\theta}{\sin\theta}\cdot\dfrac{\cos\theta}{\cos\theta}$
$= \dfrac{\sin^2\theta}{\cos\theta\sin\theta} + \dfrac{\cos^2\theta}{\cos\theta\sin\theta}$
$= \dfrac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}$
$= \dfrac{1}{\cos\theta\sin\theta}$
$= \dfrac{1}{\cos\theta}\cdot\dfrac{1}{\sin\theta}$
$= \sec\theta\csc\theta$

13. $\dfrac{\tan\theta\cot\theta}{\csc\theta} = \dfrac{\dfrac{\sin\theta}{\cos\theta}\cdot\dfrac{\cos\theta}{\sin\theta}}{\dfrac{1}{\sin\theta}}$
$= \dfrac{1}{\dfrac{1}{\sin\theta}}$
$= 1 \div \dfrac{1}{\sin\theta}$
$= 1\cdot\dfrac{\sin\theta}{1}$
$= \sin\theta$

14. $\dfrac{\cos\theta\sec\theta}{\cot\theta} = \dfrac{\dfrac{\cos\theta}{1}\cdot\dfrac{1}{\cos\theta}}{\dfrac{\cos\theta}{\sin\theta}}$
$= \dfrac{1}{\dfrac{\cos\theta}{\sin\theta}}$
$= 1 \div \dfrac{\cos\theta}{\sin\theta}$
$= 1\cdot\dfrac{\sin\theta}{\cos\theta}$
$= \tan\theta$

15. $\sin^2\theta(1+\cot^2\theta) = \sin^2\theta(\csc^2\theta)$
$= \sin^2\theta\cdot\dfrac{1}{\sin^2\theta}$
$= 1$

16. $\cos^2\theta(1+\tan^2\theta) = \cos^2\theta(\sec^2\theta)$
$= \cos^2\theta\cdot\dfrac{1}{\cos^2\theta}$
$= 1$

17. $\dfrac{1-\cos^2 t}{\cos t} = \dfrac{\sin^2 t}{\cos t}$
$= \sin t\cdot\dfrac{\sin t}{\cos t}$
$= \sin t\tan t$

18. $\dfrac{1-\sin^2 t}{\sin t} = \dfrac{\cos^2 t}{\sin t}$
$= \cos t\cdot\dfrac{\cos t}{\sin t}$
$= \cos t\cot t$

19. $\dfrac{\csc^2 t}{\cot t} = \dfrac{\dfrac{1}{\sin^2 t}}{\dfrac{\cos t}{\sin t}}$
$= \dfrac{1}{\sin^2 t} \div \dfrac{\cos t}{\sin t}$
$= \dfrac{1}{\sin^2 t}\cdot\dfrac{\sin t}{\cos t}$
$= \dfrac{1}{\sin t}\cdot\dfrac{1}{\cos t}$
$= \csc t\sec t$

20. $$\frac{\sec^2 t}{\tan t} = \frac{\frac{1}{\cos^2 t}}{\frac{\sin t}{\cos t}}$$
$$= \frac{1}{\cos^2 t} \div \frac{\sin t}{\cos t}$$
$$= \frac{1}{\cos^2 t} \cdot \frac{\cos t}{\sin t}$$
$$= \frac{1}{\cos t} \cdot \frac{1}{\sin t}$$
$$= \sec t \csc t$$

21. $$\frac{\tan^2 t}{\sec t} = \frac{\sec^2 t - 1}{\sec t}$$
$$= \frac{\sec^2 t}{\sec t} - \frac{1}{\sec t}$$
$$= \sec t - \cos t$$

22. $$\frac{\cot^2 t}{\csc t} = \frac{\csc^2 t - 1}{\csc t}$$
$$= \frac{\csc^2 t}{\csc t} - \frac{1}{\csc t}$$
$$= \csc t - \sin t$$

23. $$\frac{1-\cos\theta}{\sin\theta} = \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}$$
$$= \csc\theta - \cot\theta$$

24. $$\frac{1-\sin\theta}{\cos\theta} = \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$$
$$= \sec\theta - \tan\theta$$

25. $$\frac{\sin t}{\csc t} + \frac{\cos t}{\sec t} = \frac{\sin t}{\frac{1}{\sin t}} + \frac{\cos t}{\frac{1}{\cos t}}$$
$$= \sin t \div \frac{1}{\sin t} + \cos t \div \frac{1}{\cos t}$$
$$= \sin t \cdot \frac{\sin t}{1} + \cos t \cdot \frac{\cos t}{1}$$
$$= \sin^2 t + \cos^2 t$$
$$= 1$$

26. $$\frac{\sin t}{\tan t} + \frac{\cos t}{\cot t} = \frac{\sin t}{\frac{\sin t}{\cos t}} + \frac{\cos t}{\frac{\cos t}{\sin t}}$$
$$= \sin t \div \frac{\sin t}{\cos t} + \cos t \div \frac{\cos t}{\sin t}$$
$$= \sin t \cdot \frac{\cos t}{\sin t} + \cos t \cdot \frac{\sin t}{\cos t}$$
$$= \cos t + \sin t$$
$$= \sin t + \cos t$$

27. $$\tan t + \frac{\cos t}{1+\sin t}$$
$$= \frac{\sin t}{\cos t} + \frac{\cos t}{1+\sin t}$$
$$= \frac{\sin t}{\cos t} \cdot \frac{1+\sin t}{1+\sin t} + \frac{\cos t}{1+\sin t} \cdot \frac{\cos t}{\cos t}$$
$$= \frac{\sin t + \sin^2 t}{\cos t(1+\sin t)} + \frac{\cos^2 t}{\cos t(1+\sin t)}$$
$$\frac{\sin t + \sin^2 t + \cos^2 t}{\cos t(1+\sin t)}$$
$$= \frac{1+\sin t}{\cos t(1+\sin t)}$$
$$= \frac{1}{\cos t}$$
$$= \sec t$$

28. $$\cot t + \frac{\sin t}{1+\cos t} = \frac{\cos t}{\sin t} + \frac{\sin t}{1+\cos t}$$
$$= \frac{\cos t}{\sin t} \cdot \frac{1+\cos t}{1+\cos t} + \frac{\sin t}{1+\cos t} \cdot \frac{\sin t}{\sin t}$$
$$= \frac{\cos t + \cos^2 t}{\sin t(1+\cos t)} + \frac{\sin^2 t}{\sin t(1+\cos t)}$$
$$= \frac{\cos t + \cos^2 t + \sin^2 t}{\sin t(1+\cos t)}$$
$$= \frac{\cos t + 1}{\sin t(1+\cos t)}$$
$$= \frac{1}{\sin t}$$
$$= \csc t$$

29. $$\begin{aligned}1-\frac{\sin^2 x}{1+\cos x} &= 1-\frac{\sin^2 x}{1+\cos x}\cdot\frac{1-\cos x}{1-\cos x}\\ &= 1-\frac{\sin^2 x(1-\cos x)}{1-\cos^2 x}\\ &= 1-\frac{\sin^2 x(1-\cos x)}{\sin^2 x}\\ &= 1-1+\cos x\\ &= \cos x\end{aligned}$$

30. $$\begin{aligned}1-\frac{\cos^2 x}{1+\sin x} &= 1-\frac{\cos^2}{1+\sin x}\cdot\frac{1-\sin x}{1-\sin x}\\ &= 1-\frac{\cos^2 x(1-\sin x)}{1-\sin^2 x}\\ &= 1-\frac{\cos^2 x(1-\sin x)}{\cos^2 x}\\ &= 1-1+\sin x\\ &= \sin x\end{aligned}$$

31. $$\begin{aligned}&\frac{\cos x}{1-\sin x}+\frac{1-\sin x}{\cos x}\\ &=\frac{\cos x}{1-\sin x}\cdot\frac{1+\sin x}{1+\sin x}+\frac{1-\sin x}{\cos x}\\ &=\frac{\cos x(1+\sin x)}{1-\sin^2 x}+\frac{1-\sin x}{\cos x}\\ &==\frac{\cos x(1+\sin x)}{\cos^2 x}+\frac{1-\sin x}{\cos x}\\ &=\frac{1+\sin x}{\cos x}+\frac{1-\sin x}{\cos x}\\ &=\frac{2}{\cos x}\\ &=2\cdot\frac{1}{\cos x}\\ &=2\sec x\end{aligned}$$

32. $$\begin{aligned}&\frac{\sin x}{\cos x+1}+\frac{\cos x-1}{\sin x}\\ &=\frac{\sin x}{\cos x+1}\cdot\frac{\cos x-1}{\cos x-1}+\frac{\cos x-1}{\sin x}\\ &=\frac{\sin x(\cos x-1)}{\cos^2 x-1}+\frac{\cos x-1}{\sin x}\\ &=\frac{\sin x(\cos x-1)}{-\sin^2 x}+\frac{\cos x-1}{\sin x}\\ &=\frac{\sin x(1-\cos x)}{\sin^2 x}+\frac{\cos x-1}{\sin x}\\ &=\frac{1-\cos x}{\sin x}+\frac{\cos x-1}{\sin x}\\ &=\frac{0}{\sin x}\\ &=0\end{aligned}$$

33. $$\begin{aligned}\sec^2 x\csc^2 x &= (1+\tan^2 x)\csc^2 x\\ &= \csc^2 x+\tan^2 x\csc^2 x\\ &= \csc^2 x+\frac{\sin^2 x}{\cos^2 x}\cdot\frac{1}{\sin^2 x}\\ &= \csc^2 x+\frac{1}{\cos^2 x}\\ &= \csc^2 x+\sec^2 x\\ &= \sec^2 x+\csc^2 x\end{aligned}$$

34. $$\begin{aligned}\csc^2 x\sec x &= (1+\cot^2 x)\sec x\\ &= \sec x+\cot^2 x\sec x\\ &= \sec x+\frac{\cos^2 x}{\sin^2 x}\cdot\frac{1}{\cos x}\\ &= \sec x+\frac{\cos x}{\sin^2 x}\\ &= \sec x+\frac{1}{\sin x}\cdot\frac{\cos x}{\sin x}\\ &= \sec x+\csc x\cot x\end{aligned}$$

35. $$\frac{\sec x-\csc x}{\sec x+\csc x}=\frac{\frac{1}{\cos x}-\frac{1}{\sin x}}{\frac{1}{\cos x}+\frac{1}{\sin x}}$$
$$=\frac{\frac{1}{\cos x}-\frac{1}{\sin x}}{\frac{1}{\cos x}+\frac{1}{\sin x}}\cdot\frac{\sin x}{\sin x}$$
$$=\frac{\frac{\sin x}{\cos x}-1}{\frac{\sin x}{\cos x}+1}$$
$$=\frac{\tan x-1}{\tan x+1}$$

36. $$\frac{\csc x-\sec x}{\csc x+\sec x}=\frac{\frac{1}{\sin x}-\frac{1}{\cos x}}{\frac{1}{\sin x}+\frac{1}{\cos x}}$$
$$=\frac{\frac{1}{\sin x}-\frac{1}{\cos x}}{\frac{1}{\sin x}+\frac{1}{\cos x}}\cdot\frac{\cos x}{\cos x}$$
$$=\frac{\frac{\cos x}{\sin x}-1}{\frac{\cos x}{\sin x}+1}$$
$$=\frac{\cot x-1}{\cot x+1}$$

37. $$\frac{\sin^2 x-\cos^2 x}{\sin x+\cos x}=\frac{(\sin x+\cos x)(\sin x-\cos x)}{\sin x+\cos x}$$
$$=\sin x-\cos x$$

38. $$\frac{\tan^2 x-\cot^2 x}{\tan x+\cot x}=\frac{(\tan x-\cot x)(\tan x+\cot x)}{\tan x+\cot x}$$
$$=\tan x-\cot x$$

39. $$\tan^2 2x+\sin^2 2x+\cos^2 2x=\tan^2 2x+1$$
$$=\sec^2 2x$$

40. $$\cot^2 2x+\cos^2 2x+\sin^2 2x=\cot^2 2x+1$$
$$=\csc^2 2x$$

41. $$\frac{\tan 2\theta+\cot 2\theta}{\csc 2\theta}=\frac{\frac{\sin 2\theta}{\cos 2\theta}+\frac{\cos 2\theta}{\sin 2\theta}}{\frac{1}{\sin 2\theta}}$$
$$=\frac{\frac{\sin 2\theta}{\cos 2\theta}\cdot\frac{\sin 2\theta}{\sin 2\theta}+\frac{\cos 2\theta}{\sin 2\theta}\cdot\frac{\cos 2\theta}{\cos 2\theta}}{\frac{1}{\sin 2\theta}}$$
$$=\frac{\frac{\sin^2 2\theta+\cos 2\theta}{\cos 2\theta\sin 2\theta}}{\frac{1}{\sin 2\theta}}$$
$$=\frac{1}{\cos 2\theta\sin 2\theta}\div\frac{1}{\sin 2\theta}$$
$$=\frac{1}{\cos 2\theta\sin 2\theta}\cdot\frac{\sin 2\theta}{1}$$
$$=\frac{1}{\cos 2\theta}=\sec 2\theta$$

42. $$\frac{\tan 2\theta+\cot 2\theta}{\sec 2\theta}=\frac{\frac{\sin 2\theta}{\cos 2\theta}+\frac{\cos 2\theta}{\sin 2\theta}}{\frac{1}{\cos 2\theta}}$$
$$=\frac{\frac{\sin 2\theta}{\cos 2\theta}\cdot\frac{\sin 2\theta}{\sin 2\theta}+\frac{\cos 2\theta}{\sin 2\theta}\cdot\frac{\cos 2\theta}{\cos 2\theta}}{\frac{1}{\cos 2\theta}}$$
$$=\frac{\frac{\sin^2 2\theta}{\cos 2\theta\sin 2\theta}+\frac{\cos^2 2\theta}{\cos 2\theta\sin 2\theta}}{\frac{1}{\cos 2\theta}}$$
$$=\frac{\frac{\sin^2 2\theta+\cos^2 2\theta}{\cos 2\theta\sin 2\theta}}{\frac{1}{\cos 2\theta}}$$
$$=\frac{1}{\cos 2\theta\sin 2\theta}\div\frac{1}{\cos 2\theta}$$
$$=\frac{1}{\cos 2\theta\sin 2\theta}\cdot\frac{\cos 2\theta}{1}$$
$$=\frac{1}{\sin 2\theta}$$
$$=\csc 2\theta$$

43.
$$\frac{\tan x+\tan y}{1-\tan x\tan y}=\frac{\dfrac{\sin x}{\cos x}+\dfrac{\sin y}{\cos y}}{1-\dfrac{\sin x}{\cos y}\cdot\dfrac{\sin y}{\cos y}}\cdot\frac{\cos x\cos y}{\cos x\cos y}$$
$$=\frac{\sin x\cos y+\cos x\sin y}{\cos x\cos y-\sin x\sin y}$$

44.
$$\frac{\cot x+\cot y}{1-\cot x\cot y}=\frac{\dfrac{\cos x}{\sin x}+\dfrac{\cos y}{\sin y}}{1-\dfrac{\cos x}{\sin x}\cdot\dfrac{\cos y}{\sin y}}$$
$$=\frac{\dfrac{\cos x}{\sin x}+\dfrac{\cos y}{\sin y}}{1-\dfrac{\cos x}{\sin x}\cdot\dfrac{\cos y}{\sin y}}\cdot\frac{\sin x\sin y}{\sin x\sin y}$$
$$=\frac{\dfrac{\cos x}{\sin x}\cdot\dfrac{\sin x\sin y}{1}+\dfrac{\cos y}{\sin y}\cdot\dfrac{\sin x\sin y}{1}}{\sin x\sin y-\dfrac{\cos x}{\sin x}\cdot\dfrac{\cos y}{\sin y}\cdot\dfrac{\sin x\sin y}{1}}$$
$$=\frac{\cos x\sin y+\sin x\cos y}{\sin x\sin y-\cos x\cos y}$$

45. Left side:
$$(\sec x-\tan x)^2=\left(\frac{1}{\cos x}-\frac{\sin x}{\cos x}\right)^2$$
$$=\left(\frac{1-\sin x}{\cos x}\right)^2$$
$$=\frac{(1-\sin x)^2}{\cos^2 x}$$

Right side:
$$\frac{1-\sin x}{1+\sin x}=\frac{1-\sin x}{1+\sin x}\cdot\frac{1-\sin x}{1-\sin x}$$
$$=\frac{(1-\sin x)^2}{1-\sin^2 x}$$
$$=\frac{(1-\sin x)^2}{\cos^2 x}$$

The identity is verified because both sides are equal to $\dfrac{(1-\sin x)^2}{\cos^2 x}$.

46. Left side: $(\csc x-\cot x)^2=\left(\dfrac{1}{\sin x}-\dfrac{\cos x}{\sin x}\right)^2=\left(\dfrac{1-\cos x}{\sin x}\right)^2=\dfrac{(1-\cos x)^2}{\sin^2 x}$

Right side: $\dfrac{1-\cos x}{1+\cos x}=\dfrac{1-\cos x}{1+\cos x}\cdot\dfrac{1-\cos x}{1-\cos x}=\dfrac{(1-\cos x)^2}{1-\cos^2 x}=\dfrac{(1-\cos x)^2}{\sin^2 x}$

The identity is verified because both sides are equal to $\dfrac{(1-\cos x)^2}{\sin^2 x}$.

47. $$\frac{\tan t}{\sec t-1}=\frac{\tan t}{\sec t-1}\cdot\frac{\sec t+1}{\sec t+1}$$
$$=\frac{\tan t(\sec t+1)}{\sec^2 t-1}$$
$$=\frac{\tan t(\sec t+1)}{\tan^2 t}$$
$$=\frac{\sec t+1}{\tan t}$$

48. $$\frac{\cot t}{\csc t+1}=\frac{\cot t}{\csc t+1}\cdot\frac{\csc t-1}{\csc t-1}$$
$$=\frac{\cot t(\csc t-1)}{\csc^2 t-1}$$
$$=\frac{\cot t(\csc t-1)}{\cot^2 t}$$
$$=\frac{\csc t-1}{\cot t}$$

49. Left side:
$$\frac{1+\cos t}{1-\cos t}=\frac{1+\cos t}{1-\cos t}\cdot\frac{1+\cos t}{1+\cos t}$$
$$=\frac{(1+\cos t)^2}{1-\cos^2 t}$$
$$=\frac{(1+\cos t)^2}{\sin^2 t}$$
Right side:
$$(\csc t+\cot t)^2=\left(\frac{1}{\sin t}+\frac{\cos t}{\sin t}\right)^2$$
$$=\left(\frac{1+\cos t}{\sin t}\right)^2$$
$$=\frac{(1+\cos t)^2}{\sin^2 t}$$
The identity is verified because both sides are equal to $\frac{(1+\cos t)^2}{\sin^2 t}$.

50. Left side:
$$\frac{\cos^2 t+4\cos t+4}{\cos t+2}=\frac{(\cos t+2)(\cos t+2)}{\cos t+2}$$
$$=\cos t+2$$
Right side:
$$\frac{2\sec t+1}{\sec t}=\frac{2\sec t}{\sec t}+\frac{1}{\sec t}$$
$$=2+\cos t$$
$$=\cos t+2$$
The identity is verified because both sides are equal to $\cos t+2$.

51. $$\cos^4 t-\sin^4 t=\left(\cos^2 t-\sin^2 t\right)\left(\cos^2 t+\sin^2 t\right)$$
$$=\left(\cos^2 t-\sin^2 t\right)\cdot 1$$
$$=1-\sin^2 t-\sin^2 t$$
$$=1-2\sin^2 t$$

52. $$\frac{\sin\theta-\cos\theta}{\sin\theta}+\frac{\cos\theta-\sin\theta}{\cos\theta}$$
$$=\frac{(\sin\theta-\cos\theta)\cos\theta}{\cos\theta\sin\theta}+\frac{(\cos\theta-\sin\theta)\sin\theta}{\cos\theta\sin\theta}$$
$$=\frac{\sin\theta\cos\theta-\cos^2\theta+\sin\theta\cos\theta-\sin^2\theta}{\sin\theta\cos\theta}$$
$$=\frac{2\sin\theta\cos\theta-\left(\cos^2\theta+\sin^2\theta\right)}{\sin\theta\cos\theta}$$
$$=\frac{2\sin\theta\cos\theta-1}{\sin\theta\cos\theta}$$
$$=\frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta}-\frac{1}{\sin\theta\cos\theta}$$
$$=2-\frac{1}{\sin\theta}\cdot\frac{1}{\cos\theta}$$
$$=2-\csc\theta\sec\theta$$
$$=2-\sec\theta\csc\theta$$

54. $\dfrac{\sin\theta}{1-\cot\theta}-\dfrac{\cos\theta}{\tan\theta-1}$

$=\dfrac{\sin\theta}{1-\dfrac{\cos\theta}{\sin\theta}}-\dfrac{\cos\theta}{\dfrac{\sin\theta}{\cos\theta}-1}$

$=\dfrac{\sin\theta}{\dfrac{\sin\theta}{\sin\theta}-\dfrac{\cos\theta}{\sin\theta}}-\dfrac{\cos\theta}{\dfrac{\sin\theta}{\cos\theta}-\dfrac{\cos\theta}{\cos\theta}}$

$=\dfrac{\sin\theta}{\dfrac{\sin\theta-\cos\theta}{\sin\theta}}-\dfrac{\cos\theta}{\dfrac{\sin\theta-\cos\theta}{\cos\theta}}$

$=\sin\theta\div\dfrac{\sin\theta-\cos\theta}{\sin\theta}-\cos\theta\div\dfrac{\sin\theta-\cos\theta}{\cos\theta}$

$=\sin\theta\cdot\dfrac{\sin\theta}{\sin\theta-\cos\theta}-\cos\theta\cdot\dfrac{\cos\theta}{\sin\theta-\cos\theta}$

$=\dfrac{\sin^2\theta}{\sin\theta-\cos\theta}-\dfrac{\cos^2\theta}{\sin\theta-\cos\theta}$

$=\dfrac{\sin^2\theta-\cos^2\theta}{\sin\theta-\cos\theta}$

$\dfrac{(\sin\theta-\cos\theta)(\sin\theta+\cos\theta)}{\sin\theta-\cos\theta}$

$=\sin\theta+\cos\theta$

55. $\left(\tan^2\theta+1\right)\left(\cos^2\theta+1\right)$

$=\tan^2\theta\cos^2\theta+\tan^2\theta+\cos^2\theta+1$

$=\dfrac{\sin^2\theta}{\cos^2\theta}\cdot\cos^2\theta+\tan^2\theta+\cos^2\theta+1$

$=\sin^2\theta+\tan^2\theta+\cos^2\theta+1$

$=\sin^2\theta+\cos^2\theta+\tan^2\theta+1$

$=1+\tan^2\theta+1$

$=\tan^2\theta+2$

56. $(\cot^2\theta+1)(\sin^2+1)$

$=\cot^2\theta\sin^2\theta+\cot^2\theta+\sin^2\theta+1$

$=\dfrac{\cos^2\theta}{\sin^2\theta}\sin^2\theta+\cot^2\theta+\sin^2\theta+1$

$=\cos^2\theta+\sin^2\theta+\cot^2\theta+1$

$=1+\cot^2\theta+1$

$=\cot^2\theta+2$

57. $(\cos\theta-\sin\theta)^2+(\cos\theta+\sin\theta)^2=\cos^2\theta-2\cos\theta\sin\theta+\sin^2\theta+\cos^2\theta+2\cos\theta\sin\theta+\sin^2\theta$

$=\cos^2\theta+\sin^2\theta+\cos^2\theta+\sin^2\theta$

$=1+1=2$

58. $(3\cos\theta-4\sin\theta)^2+(4\cos\theta+3\sin\theta)^2$

$=9\cos^2\theta-24\cos\theta\sin\theta+16\sin^2\theta+$

$+16\cos^2\theta+24\cos\theta\sin\theta+9\sin^2\theta$

$=9\cos^2\theta+9\sin^2\theta+16\sin^2\theta+16\cos^2\theta$

$=9(\cos^2\theta+\sin^2\theta)+16(\sin^2\theta+\cos^2\theta)$

$=9(1)+16(1)$

$=25$

59. $\dfrac{\cos^2x-\sin^2x}{1-\tan^2x}=\dfrac{\cos^2x-\sin^2x}{1-\dfrac{\sin^2x}{\cos^2x}}=\dfrac{\cos^2x-\sin^2x}{\dfrac{\cos^2x-\sin^2x}{\cos^2x}}$

$=\dfrac{\cos^2x-\sin^2x}{1}\div\dfrac{\cos^2x-\sin^2x}{\cos^2x}$

$=\dfrac{\cos^2x-\sin^2x}{1}\cdot\dfrac{\cos^2x}{\cos^2x-\sin^2x}=\cos^2x$

60. $$\frac{\sin x+\cos x}{\sin x}-\frac{\cos x-\sin x}{\cos x}$$
$$=\frac{(\sin x+\cos x)\cos x}{\sin x\cos x}-\frac{(\cos x-\sin x)\sin x}{\sin x\cos x}$$
$$=\frac{\sin x\cos x+\cos^2 x-\cos x\sin x+\sin^2 x}{\sin x\cos x}$$
$$=\frac{\cos^2 x+\sin^2 x}{\sin x\cos x}$$
$$=\frac{1}{\sin x\cos x}$$
$$=\frac{1}{\sin x}\cdot\frac{1}{\cos x}$$
$$=\csc x\sec x$$
$$=\sec x\csc x$$

61. Conjecture: left side is equal to $\cos x$
$$\frac{(\sec x+\tan x)(\sec x-\tan x)}{\sec x}=\frac{\sec^2 x-\tan^2 x}{\sec x}$$
$$=\frac{1}{\sec x}$$
$$=\cos x$$

62. Conjecture: left side is equal to $\sin x$
$$\frac{\sec^2 x\csc x}{\sec^2 x+\csc^2 x}=\frac{\frac{1}{\cos^2 x}\cdot\frac{1}{\sin x}}{\frac{1}{\cos^2 x}+\frac{1}{\sin^2 x}}\cdot\frac{\cos^2 x\sin^2 x}{\cos^2 x\sin^2 x}$$
$$=\frac{\sin x}{\sin^2 x+\cos^2 x}$$
$$=\frac{\sin x}{1}$$
$$=\sin x$$

63. Conjecture: left side is equal to $2\sin x$
$$\frac{\cos x+\cot x\sin x}{\cot x}=\frac{\cos x}{\cot x}+\frac{\cot x\sin x}{\cot x}$$
$$=\frac{\cos x}{\frac{\cos x}{\sin x}}+\frac{\cot x\sin x}{\cot x}$$
$$=\frac{\cos x\sin x}{\cos x}+\sin x$$
$$=\sin x+\sin x$$
$$=2\sin x$$